Falaco Solitons, Cosmological strings in swimming pool.
Update of arXiv/gr-qc/0101098

R. M. Kiehn
Emeritus Professor of Physics
University of Houston

http://www.cartan.pair.com

June 13, 2006

1. Preface

This monograph is a condensed version (with almost no math) describing the Falaco Solitons. It was a serendipitous event that brought these objects to my attention, almost twenty years ago (1986). More detail (and more mathematics) can be found in chapter 2 of the second volume [42] in a series in which topological methods are applied to the study of diverse Non Equilibrium Systems and Irreversible Processes. The focus herein is on the experimental properties and significance of the Falaco Solitons, which are, in effect, extraordinary topological defect structures that can be created in a swimming pool. These Falaco Solitons turn out to be locally unstable, but globally stabilized, long lived, topologically coherent strucutures, that are far from equilibrium. The appearance of these topological defect structures appears to be universal, from the quantum microscopic domain of Bose-Einstein condensates to the Cosmics strings between Galaxies.

The monograph starts with the experimental observation that highly motivated and sustained the author's research interest in Non Equilibrium Systems and Irreversible Processes. The experiment is easily performed (and has won prizes at state fairs for science projects conducted by high school students in the USA). Chapter 1 goes directly to a discussion of the extraordinary topological defects (known as Falaco Solitons) that can be (and have been) created and studied in a swimming pool. The ability to create Falaco Solitons gives a high level of credence to the fundamental theory of continuous topological evolution. The Falaco solitons appear to have properties claimed by the String theorists who are trying to explain Quantum Gravity. The gauntlet was thrown down to several "String Theorists", challenging them to solve the problems related to the formation, universality, and persistence of the Falaco solitons, using their mathematical conjectures. There have been no replies as of December 1, 2005. The elite group of String theorists seem to ignore both the challenge and the experimental facts.

2. Cosmic Strings in a Swimming Pool

During March of 1986, while visiting an old MIT friend in Rio de Janeiro, Brazil, the present author became aware of a significant topological event involving visual solitons that can be replicated experimentally by almost everyone with access to a swimming pool. Study the photo which was taken by David Radabaugh, in the late afternoon, Houston, TX 1986.

Figure 2.1. Three Falaco Soliton pairs in a Swimming Pool

The extraordinary photo is an image of 3 pairs of what are now called Falaco Solitons, a few minutes after their creation. Each Falaco Soliton consists of a pair of globally stabilized rotational indentations in the water-air density discontinuity surface of the swimming pool. The dimple shape is as if a conical pencil point was pushed into a rubber sheet, causing a deformation; but the indentation is dominated by dynamic intransitive rotation, not transitive affine translation. Unseen in the photograph, each pair of contra-rotating dimples are connected by a singular thread in the form of a circular arc extending from the vertex of one dimple to the vertex of the other dimple of the pair. The "thread" can be made "visible" by injecting drops of dye into the fluid near the rotation axis of one of the dimples. These Solitons are apparently long-lived states of matter far from thermodynamic equilibrium. They will persist for many minutes in a still pool of water, maintaining their topological coherence so as to permit their inclusion into the class of objects called Solitons. The Falaco Solitons are extraordinary, not only due to the fact that they are so easily created in a macroscopic dynamical systems environment, but also because they offer real life, easily observed, evidence for the continuous evolution and creation of topological defects.

The long lifetime and the topological coherence stability of the Falaco Solitons in a dissipative fluid media is not only remarkable, but also is a matter of applied theoretical interest. The equilibrium discontinuity surface of the fluid in the "uniform" gravitational field is flat, and has both zero mean curvature and zero Gauss curvature. The shape of the observed discontinuity surface defect of a Falaco Soliton dimple indicates that the surface mean curvature is zero, but the Gauss curvature is not zero. In Euclidean spaces, such real surfaces of xero Mean curvature are *minimal* surfaces of *negative* Gauss curvature. Such surfaces are locally unstable, so it has been presumed that the pair of defect structures that make up the Falaco Soliton must be globally stabilized. It has been conjectured that the connecting string is under tension in order to maintain the shape of the pair of dimpled indentations. This conjecture is justified by the observation that if the singular thread is abruptly "severed" (by experimental "chopping motions" under the surface of the fluid), the dimpled endcaps disappear in a rapid, non-diffusive, manner. However, Euclidean minimal surfaces, of negative Gauss curvature, do not have a conical singularity that appears to be a property of the Falaco Solition. The conical singularity offers a connection point for the "string" that seems to be attached to each of the Falaco vertices. As shown below, it appears that the conical dimple of zero mean curvature and *positive* Gauss curvature can be achieved from the analysis of minimal surfaces in a Non-Euclidean space. In such an analysis, the curvatures are equal and opposite pure imaginary numbers whose sum produces a zero Mean curvature, but whose product produces a Gaussian curvature which is real and positive. The imaginary curvatures are signatures of rotational, not translational, effects. Rotations seem to be characteristic of the Falaco Soliton pairs. Such rotational concepts can be asociated with what the mathematicians have defined as Spinors.

The dimpled surface pairs of the Falaco Soliton are most easily observed in terms of the dramatic black discs that they create by projection of the solar rays to the bottom of the pool. The optics of this effect will be described below. Careful examination of the photo of Figure 1 will indicate, by accidents of noticeable contrast and reflection, the region of the dimpled surface of circular rotation. The dimples appear as (deformed) artifacts to the left of each black spot, and elevated above the horizontal plane by about 25 degrees (as the photo was taken in late afternoon). Also, notice that the vestiges of caustic spiral arms in the surface structures around each pair of rotation axes can be seen. These surface spiral arms can be visually enhanced by spreading chalk dust on the free surface of the pool. The bulk fluid motion is a local (non-rigid body) rotational motion about the interconnecting circular thread. In the photos of Figure 1 and Figure 2, the depth of each of the actual indentations of the free surface is, at most, of a few millimeters in extent.

A better photo, also taken by D. Radabaugh, but in the year 2004 in a swimming pool in Mazan, France, demonstrates more clearly the dimpled surface defects, and the Snell refraction. The sun is to the left and at an elevation of about 30 degrees.

Figure 2.2. Surface Indentations of a Falaco Soliton

The photo is in effect a single frame of a digital movie that demonstrates the creation and evolutionary motions of the Falaco Solitons. The experimental details of creating the Falaco Solitons are described below, but the movie explains their creation and dynamics far better than words. The digital movie may be downloaded from [37].

Remark 1. *It is possible to produce, hydrodynamically, in a viscous fluid with a surface of discontinuity, a long lived topologically coherent structure that consists of a set of macroscopic topological defects. The Falaco Solitons are representative of non-equilibrium long lived structures, or "stationary states", far from equilibrium.*

These observations were first reported at the 1987 Dynamics Days conference in Austin, Texas [30] and subsequently in many other places, mostly in the hydrodynamic literature [31], [33], [35], [32], as well as several APS meetings.

2.1. Falaco Surface dimples are of zero mean curvature

From a mathematical point of view, the Falaco Soliton is interpreted as a connected pair of two dimensional topological defects connected by a one dimensional topological defect or thread. The surface defects of the Falaco Soliton are observed dramatically due the formation of circular black discs on the bottom of the swimming pool. The very dark black discs are emphasized in contrast by a bright ring or halo of focused light surrounding the black disc. All of these visual effects can be explained by means of the unique optics of Snell refraction from a surface of zero mean curvature.

Remark 2. *This explanation of the optics was reached about 30 minutes after I first became aware of the Soliton structures, while standing in the pristine white marble swimming pool of an old MIT roommate, Jose Haraldo Falçao, under the brilliant Brazilian sunshine in Rio de Janeiro. At MIT, Haraldo was always called Falaco, after he scored 2 goals in a MIT soccer match, and the local newspapers misprinted his name. Hence I dubbed the topological defect structures, Falaco Solitons. Haraldo will get his place in history. I knew that finally I had found a visual, easily reproduced, experiment that could be used to show people the importance and utility of Topological Defects in the physical sciences, and could be used to promote my ideas of Continuous Topological Evolution.*

The observations were highly motivating. The experimental observation of the Falaco Solitons greatly stimulated me to continue research in applied topology, involving topological defects, and the topological evolution of such defects which can be associated with phase changes and thermodynamically irreversible and turbulent phenomena. When colleagues in the physical and engineering sciences would ask “What is a topological defect?” it was possible for me to point to something that they could replicate and understand visually at a macroscopic level.

During the initial few seconds of decay to the metastable soliton state, each large black disk is decorated with spiral arm caustics, remindful of spiral arm galaxies. The spiral arm caustics contract around the large black disk during the stabilization process, and ultimately disappear when the "topological steady" soliton state is achieved. The spiral caustics appear to be swallowed up by the black "hole". It should be noted that if chalk dust is sprinkled on the surface of the pool during the formative stages of the Falaco Soliton, then the topological signature of the familiar Mushroom Spiral pattern is exposed.

Notice that the black spots on the bottom of the pool in the photo are circular and not distorted ellipses, even though the solar elevation in the photo is less than 30 degrees. The important experimental fact deduced from the optics of Snell refraction is that each dimpled surface appears to be a surface of zero mean curvature. This conclusion is justified by the fact that the Snell projection to the floor of the pool is almost conformal, preserving the circular appearance of the black disc, independent from the angle of solar incidence. This conformal projection property of preserving the circular shape is a property of normal projection from minimal surfaces of zero mean curvature [38].

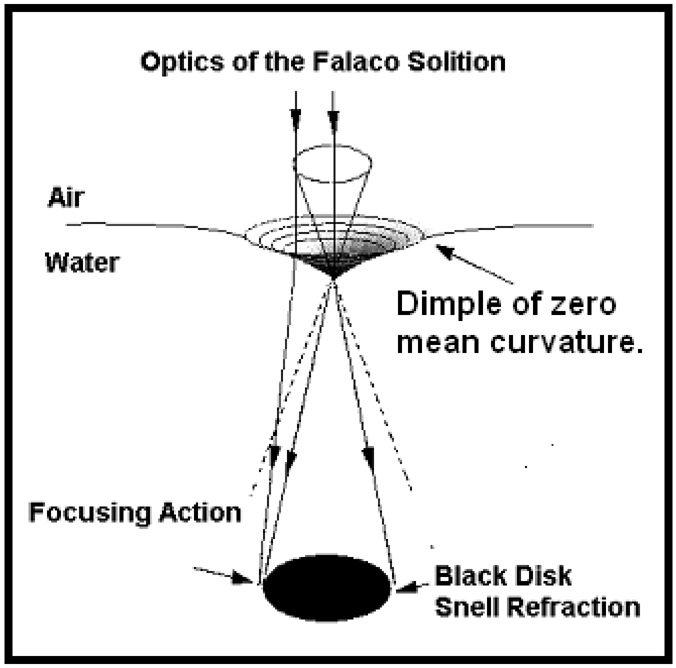


Figure 2.3. Snell Refraction of a Falaco Soliton surface defect.

As mentioned above, a feature of the Falaco Soliton [30] that is not immediately obvious is that it consists of a pair of two dimensional topological defects, in a surface of fluid discontinuity, which are *connected* by means of a topological singular thread.

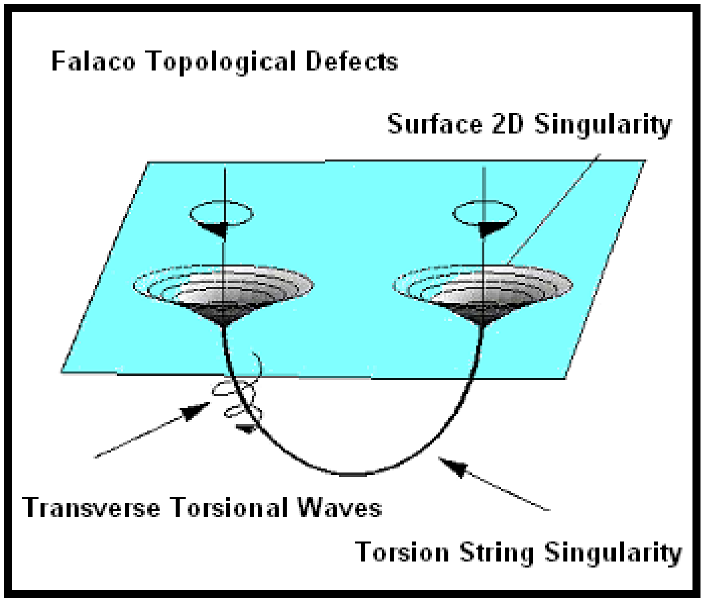


Figure 2.4. Falaco Topological Defects with connecting thread.

Dye injection near an axis of rotation during the formative stages of the Falaco Soliton indicates that there is a unseen thread, or 1-dimensional string singularity, in the form of a circular arc that connects the two 2-dimensional surface singularities or dimples. Transverse Torsional waves made visible by dye streaks (caused by dye drops injected near one of the surface rotation axes) can be observed to propagate, back and forth, from one dimple vertex to the other dimple vertex, guided by the "string" singularity. The effect is remindful of the whistler propagation of electrons along the guiding center of the earth's pole to pole magnetic field lines. However, as a soliton, the topological system retains its coherence for remarkably long time - more than 15 minutes in a still pool. The long lifetime of the Falaco Soliton is conjectured to be due to this *global stabilization* of the connecting string singularity, even though a real surface of zero mean curvature is locally unstable. The Falaco Soliton starts out from a non-equilibrium thermodynamic state of Pfaff topological dimension 4, which quickly and irreversibly decays to a "topologically stationary" state, still far from equilibrium, but with a long dynamic lifetime [41] [42].

2.1.1. Zero Mean curvature and Harmonic vector fields.

The long life of the soliton state in the presence of a viscous media indicates that the flow vector field describing the dynamics is probably harmonic. This result is in agreement with the assumption that the fluid can be represented by a Navier-Stokes equation where the viscous dissipation is dominated by affine shear viscosity times the vector Laplacian of the velocity field. If the velocity field is harmonic, the vector Laplacian vanishes, and the shear dissipation term goes to zero - no matter what is the magnitude of the shear viscosity term. Hence a palatable argument is offered in terms of harmonic velocity fields for the existence of the long lifetime of the Falaco Solitons (as well as the production of wakes in fluid dynamics [43]). More over it is known in the theory of minimal surfaces [24] that surfaces of zero mean curvature are generated by harmonic vector fields.

Remark 3. *The idea of a long lifetime in a dissipative media is to be associated with Harmonic vector fields and surfaces of zero mean curvature.*

2.1.2. Dimple Stability

In Euclidean space, the real minimal surface defects of zero mean curvature are of negative (or zero) Gauss curvature, and are, therefor, locally unstable. However, stationary non rotating soap films can be stabilized by certain boundary conditions. The soap film of one connected component - between the boundary formed by two separated coaxial rings of equal diameter - is a real minimal surface. The surface is stable only if the separation of the the rings is less than (approximately) 2.65 times the minimal throat diameter. Experimentally the *stationary non-rotating* soap film between two boundary rings will break apart if the soap film is stretched too far. The single component catenoid (with zero mean curvature and negative Gauss curvature, and with real equal and opposite principle curvatures) will bifurcate into two flat components. The two components, one on each ring, are of zero Gauss curvature as well as zero mean curvature. As mentioned above the experimental equilibrium state of the fluid discontinuity surface is a surface of both zero Gauss curvature and zero Mean curvature (both principle surface curvatures are zero). From the optics of Snell refraction, a Falaco endcap is obviously a surface of zero mean curvature, and if equivalent to a stationary soap film, it should be locally unstable. However, it was conjectured that the local instability could be overcome globally by a string whose tension globally stabilizes the locally unstable endcaps. Could the tension be related to a rotationally induced positive contribution to the otherwise negative Gauss curvature? These conjectures originally are explained (partially) in terms of a bifurcation process and solutions to the Navier-Stokes equations in a rotating frame of reference. A summary of such analysis is presented in [42].

3. Falaco Solitons as experimental artifacts

3.1. Minimal and Maximal Surfaces of Zero Mean curvature

More recently (January 2005), it was determined that an alternative, and perhaps better, description can be given in terms of a fluid with a surface discontinuity that has zero mean curvature relative to a Minkowski metric; the Minkowski surface of zero mean curvature has a Gauss curvature which is positive, not negative. A topological birfurcation process from a Rankine vortex to Falaco Solitons would then be such as to change the 3D Euclidean signature into a 3D Non-Euclidean Minkowski signature. Such surfaces of zero mean curvature embedded in Minkowski space have been called *maximal* surfaces by the differential geometers, and have conical singular points [15] . It is now believed that the Falaco thread is attached to the conical singular points of a pair of such "maximal" surfaces.

Maximal surfaces are 2D surfaces of zero mean curvature that are generated by immersive maps from a two dimension space into a 3 dimensional space with a Lorentz metric [15]. The maximal surface is defined in terms of a space-like immersion with positive Gauss curvature and with zero mean curvature. Such maximal surfaces are to be compared to minimal surfaces in a space with a Euclidean metric, but note that minimal surfaces in Euclidean space have a negative Gauss curvature as well as zero mean curvature. Maximal surfaces can admit isolated, or "conical", singularities, where Minimal surfaces do not. Maximal surfaces can mimic catenoidal and helical surfaces of Euclidean theory, but may exhibit singular subsets of points. It is remarkable (and discussed in the next section) that such maximal surfaces can appear in fluids as propagating long lived topological defects which have been described above as Falaco Solitons.

Forgive the use of of a bit of mathematics but the idea, of a physical effect having a realization in terms of a Non-Euclidean 3 dimensional geometry, is a rather dramatic surprise to most engineers and physicists. It is almost as dramatic as the discovery that the space time of electromagnetic signals is a Non-Euclidean 4D space with a Minkowski - Lorentz metric.

3.1.1. Zero Mean curvature Surfaces of Revolution in Minkowski 3-Space

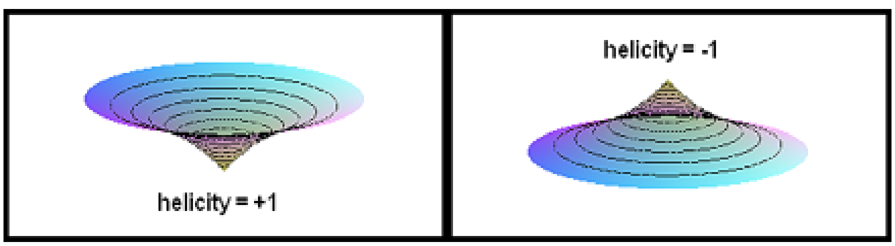
The Non-Euclidean 3 dimensional geometry has a metric (defining infinitesimal lengths) of the form

$$(ds)^2 = (dx)^2 + (dy)^2 - (dz)^2. \tag{3.1}$$

The immersion of a 2D surface with surface parameters, {u,v} into such Non-Euclidean 3D spaces can be expressed by the formula:

$$R(u,v) = [(\sinh(v)\cos(u), (p\sinh(a)\sin(u), h\ v]. \tag{3.2}$$

This formula generates a surface of zero Mean curvature (see the Figures 3.1a and 3.1b) in a space with a Minkowski metric! The coefficient p is related to the handedness of the rotation about the z axis, and h is related to the helicity along the z axis. The surface is of zero mean curvature, but the metric vanishes at the conical singular point: the Gauss curvature becomes infinite! The immersion does not generate a minimal surface in euclidean space. For other examples of zero mean curvature surfaces in both Euclidean and Minkowski spaces see [23]



Figures 3.1a. and 3.1b. Minkowski surfaces of zero mean curvature

The surface is similar to the hyperbolic minimal surface (Catenoid) in Euclidean geometry, but here, unlike the Euclidean catenoid, the Minkowski catenoid has a singular point. It would appear that this a better description of the Falaco Soliton dimple, than that given by a Euclidean catenoid. The surface is sensitive to the sign of the directional chirality ($h = \pm 1$), but is not sensitive to the handedness of polarization, p.

3.1.2. Zero Mean curvature Surfaces of Revolution in Eucidean 3-Space

The formula for a hyperbolic rotational immersion,

$$R(u,v) = [\cosh(v)\cos(u), p\cosh(v)\sin(u), hv], \tag{3.3}$$

generates a minimal surface of zero mean curvature in a space with an Euclidean metric. The surface "mimics" a Wheeler wormhole, and the soap film between two rings separated by a diameter. The zero mean curvature surface is also sensitive to the sign of the directional chirality ($h = \pm 1$), but is not sensitive to the handedness of rotational polarization, p.

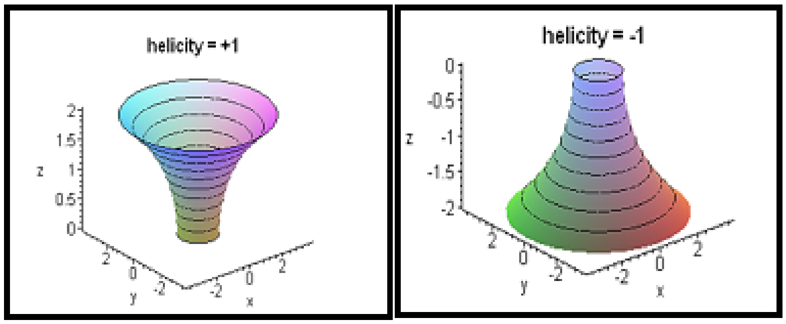


Fig. 3.2a and 3.2b. Rotational Surfaces zero mean curvature in Euclidean Space

3.1.3. Immersions that do not depend upon the 3D signature

On the other hand, the surface generated by the hyperbolic helical immersion

$$R(u, v) = [(\cosh(v)\cos(u), (p\cosh(v)\sin(u), h\ u] \tag{3.4}$$

also is a surface of zero mean curvature in *both* a Euclidean space or in Minkowski space. The helical surfaces are ruled helices rapped around a "hole" of radius unity. The Helical surface is sensitive to the sign of the product of the rotational polarization, p, and the directional chirality, h.

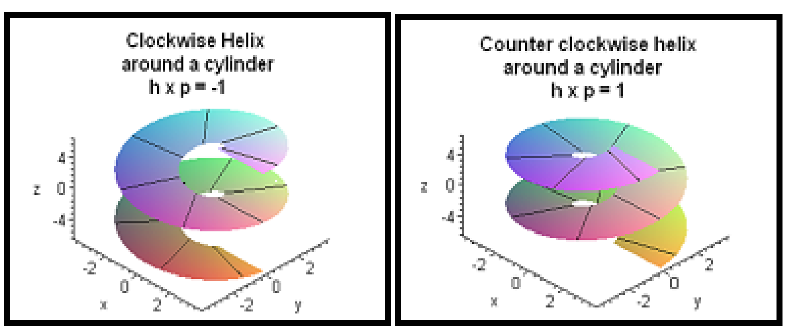


Figure 3.3a. and 3.3b. Helical Surfaces of zero mean curvature independent from 3D signature.

The Gauss curvature of the immersion is negative and bounded in Euclidean Space. The Gauss curvature of the immersion is positive and singular for $v = 0$ in the Minkowski space. Both surfaces have zero mean curvature. The principal surface curvatures are real and of opposite in sign for the Euclidean 3 space, and are pure imaginary and of opposite sign in Minkowski 3 space. In both cases the Gauss curvature is real but of different signs.

The zero mean curvature surfaces, with a singular point (as in Figures 1.5a. and 1.5b.), can be formed experimentally in a fluid. The experimental evidence has been described above. The idea that 3-dimensional space may or may not be visually Euclidean challenges a dogmatic precept of modern physics, where it is rarely perceived that physical 3D space can be anything but Euclidean. However, as discussed in the following section, the occurrence of long lived rotational structures in the free surface of a water, which have been described as Falaco Solitons, exhibit the features of maximal surfaces in a Lorentz - Minkowski space. The Falaco Solitons are topological defect structures easily replicated in an experimental sense. Optical measurements indicate that the surface defect structures have a zero mean curvature. In addition, the surface defect structures have an apparent conical singularity which is an artifact of the signature of a maximal space-like surface in Minkowski space. Maximal surfaces are generated by immersive maps from a two dimension space into a 3 dimensional space with a Lorentz metric [15]. The maximal surface is defined in terms of a space like immersion with positive Gauss curvature and with zero mean curvature. Such surfaces are related to minimal surfaces in a space with a Euclidean metric, but minimal surfaces in Euclidean space have negative Gauss curvature. Maximal Surfaces can admit isolated, or "conical", singularities, where Minimal surfaces can not. Maximal surfaces can mimic catenoidal and helical surfaces of Euclidean theory, but may exhibit singular subsets of points.

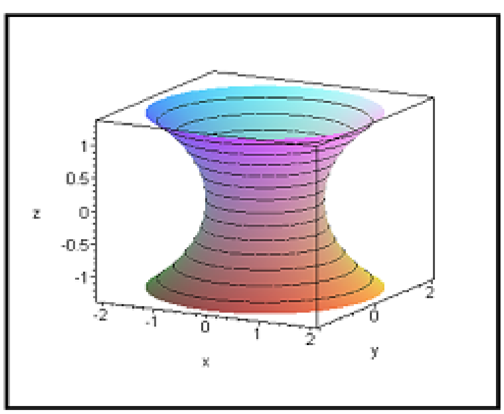
The zero mean curvature surfaces, with a singular point, can be formed experimentally in a fluid. The experimental evidence is given by the existence of the Falaco Solitons. The idea that 3-dimensional space may or may not be Euclidean challenges a dogmatic precept of modern physics, where it is rarely perceived that physical 3D space can be anything but Euclidean. However, as discussed above, the occurrence of long lived rotational structures in the free surface of a water, which have been described as Falaco Solitons, exhibit the features of maximal surfaces in a Lorentz - Minkowski space. The Falaco Solitons are topological defect structures easily replicated in an experimental sense. Optical measurements indicate that the surface defect structures have a zero mean curvature. In addition, the surface defect structures have an apparent conical singularity which is an artifact of the signature of a maximal space-like surface in Minkowski space.

The conjecture is that the Falaco Solitons are topological defects caused by the decay of a dissipative Pfaff dimension 4 domain, with a spacelike Euclidean structure, followed by a topological bifurcation process that changes the space-like Sylvester signature from a 3D Euclidean structure to a space like 3D Minkowski structure.

Alternatively, the Euclidean metric can be maintained, and a result similar to the immersion of the 2D surface into Minkowski space can be attributed to the fact that infinitesimal rotations admit Spinor complex isotropic eigen direction fields with non-zero, pure imaginary eigen values. The Gauss curvature of such systems is positive, even though the eigen direction fields are complex Spinors, not vectors in the diffeomorphic sense. A discussion of the Hopf map as applied to this idea will be found in [42].

3.2. Wheeler Wormholes and Falaco Solitons

Initially it was thought that the Falaco surface indentation, immediately after creation, was in the form of a Rankine vortex (with regions of positive mean curvature, and positive Gauss curvature in a 3D euclidean space), which then decayed (somehow) into a classic *minimal* surface of zero mean curvature, but negative Gauss curvature. Admittedly, the extended conjugate catenoids of Figure 1.8 (a deformed Wheeler Wormhole with an open throat ?) have some of the features and appearance of the Falaco Solitons. However, the extended singular thread (without an open throat) between vertex singularities does not appear to be replicated by the minimal surface of negative Gauss curvature.



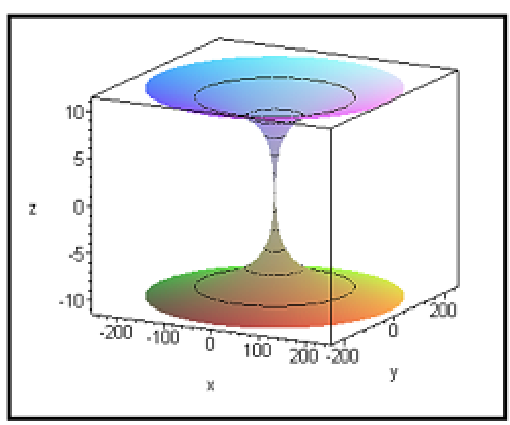


Fig. 3.4 a Soap film between rings   Fig. 3.4b Deformed Wheeler Wormhole

Evolutionary processes that are solutions of the Navier-Stokes equations of fluid dynamics *in a rotating frame of reference* seem to mimic the features of the Falaco Solitons to a certain extent. However, the soluitons to the Navier-Stokes dynamics seems to require that the connection (the string) between the Falaco dimple pairs has an open throat (like a Wheeler Worm hole).

For a "stationary" euclidean soap film between two coaxial boundary rings, the system with negative Gauss curvature is stable only if the axial separation of the boundary rings is less than (approximately) 2.65 times the minimal throat diameter. The instability of a surface of the soap film type (Wheeler Wormhole with open throat) has been demonstrated analytically for a fluid flow in a rotating frame, where the zero set of the helicity function of the fluid flow has the appearance of a real minimal surface. As the bulk flow increases, the helicity function changes sign, and therefor represents a change in topology from a connected set to a disconnected set. With the change in sign, a torsion bubble (or a torsion burst) appears in the flow pattern [33] [43]. Such torsion bursts (incorrectly called vortex bursts?) have been observed by jet pilots in extreme maneuvers.

The stability argument makes it difficult to utilize the real minimal surface "soap film" concept, with negative Gauss curvature, as a model representing the topological structure of the Falaco Soliton of two dimpled surfaces of zero mean curvature connected by a 1 dimensional thread.

Remark 4. *The bottom line is that the remarkable features of creating a stable surface of zero mean curvature and positive Gauss curvature (the Falaco Soliton) is explained either by assuming that the usual 3D Euclidean Signature is rotationally dependent and can topologically evolve into a 3D Minkowski Signature; or, the Euclidean Signature is preserved, and a macroscopic evolutionary process described by complex Spinor direction fields (which are not the same as diffeomorphic vector fields) must be admitted on thermodynamic grounds.*

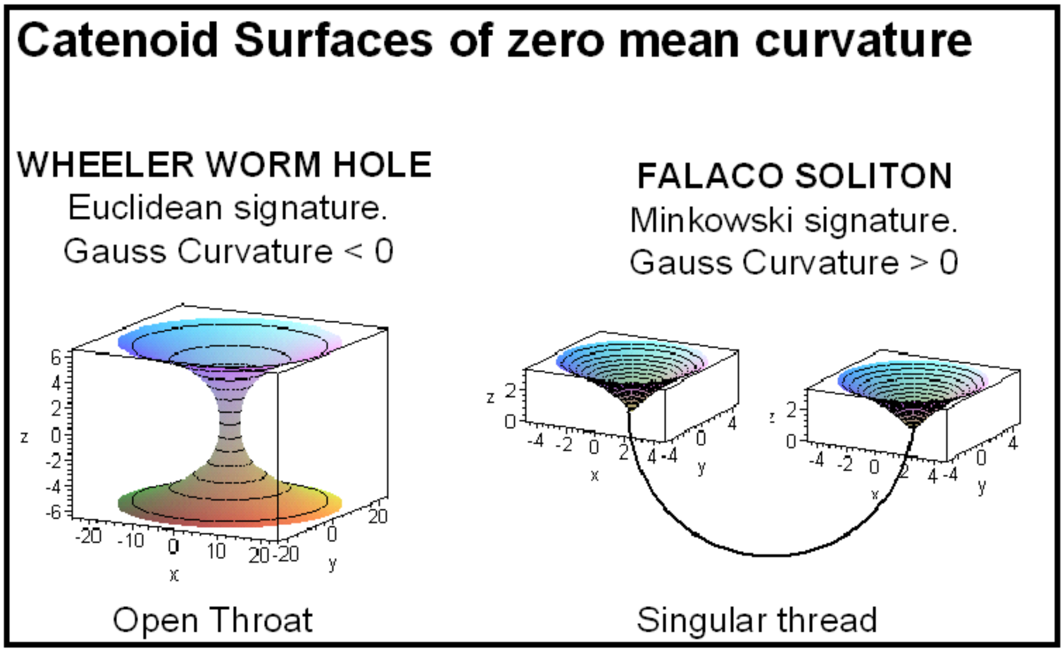


Figure 3.5 Rotational Surfaces of Zero mean curvature in Euclidean and Minkowski 3 space.

It is extraordinary, but the Falaco Solitons appear to be another form of a zero mean curvature surface structure connecting discontinuity surfaces. Such connections of one surface to another surface, or to two different regions of the same surface, either are related to macroscopic realizations of the Wheeler wormhole (soap film with possibly a very narrow, but open, throat), or to Spinor (complex) surfaces generated by complex eigen direction fields of infinitesimal rotations, producing pairs of dimpled conical zero mean curvature surfaces, connected by a 1 dimensional thread (a closed "throat").

The Wheeler wormhole structure was presented early on by Wheeler (1955), but was considered to be unattainable in a practical sense. To quote Zeldovich p. 126 [47]

> "The throat or "wormhole" (in a Kruskal metric) as Wheeler calls it, connects regions of the same physical space which are extremely remote from each other. (Zeldovich then gives a sketch that topologically is similar to the Falaco Soliton). Such a topology implies the existence of 'truly geometrodynamic objects' which are unknown

> to physics. Wheeler suggests that such objects have a bearing on the nature of elementary particles and anti particles and the relationships between them. However, this idea has not yet borne fruit; and there are no macroscopic "geometrodynamic objects" in nature that we know of. Thus we shall not consider such a possibility."

Now the experimental evidence justifies (again) Wheeler's intuition. Both the Wheeler wormhole and the Falaco Soliton are related to surface structures of zero mean curvature. The catenoidal surface of zero mean curvature, and negative Gauss curvature, in a 3D Euclidean space is a Wheeler Wormhole (with an open throat), while the conical surface of zero mean curvature, and positive Gauss curvature, and its conical singular point in a 3D Minkowski space is a part of the rotationally induced Falaco Soliton.

3.3. Falaco Solitons as Spiral Arm Galaxies

During the early stages of formation of the Falaco Solitons, caustics could be observed on the surface of density discontinuity that mimic the spiral arms so often observed in paper thin, almost "flat", galaxies such as our own Milky Way. The observation is enhanced if chalk dust (or dirt) is deposited on the surface of the pool during the first few seconds of Soliton formation. The observation is so stimulating that it leads to the conjecture that perhaps the spiral arm galaxies of the cosmos come in connected pairs. Could it be true that the Milky Way galaxy and its spiral arm companion, M31 are Falaco Solitons connected by some stabilizing thread?

Only recently has photographic evidence appeared suggesting that galaxies may be connected by strings.

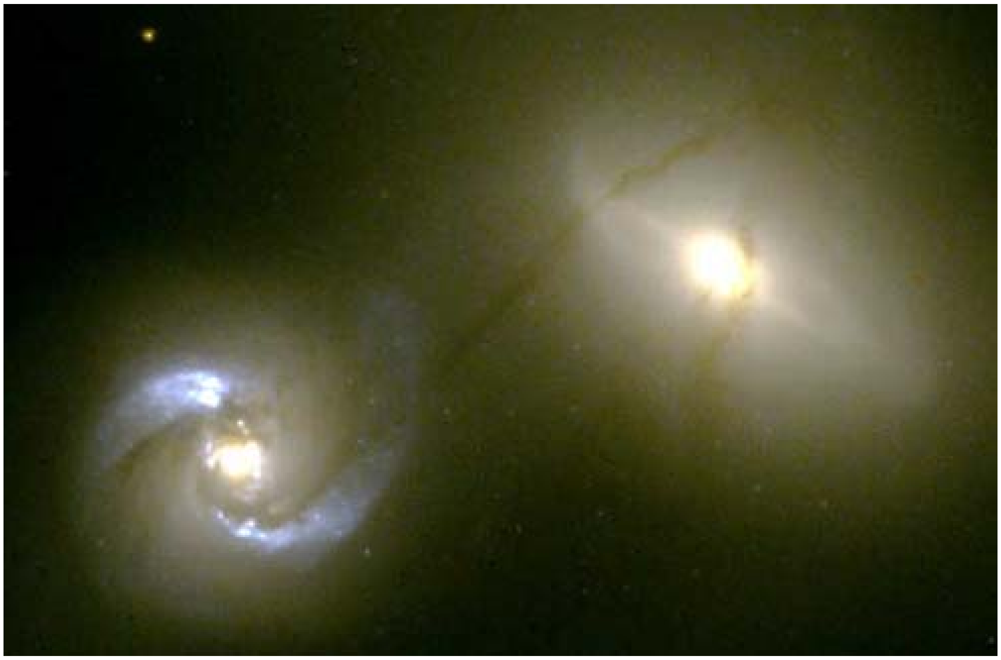


Figure 3.6

Indeed, the visual exhibition at the macroscopic level of dynamic topological coherent structures in a swimming pool, connected by a string, gives a level of credence to esoteric constructions of string theory. It is strange that the string theorists have ignored the experimental observations of Falaco Solitons where the "string" is neither microscopically small, nor folded into another dimension.

3.4. Falaco Solitons as Strings between Branes

The idea that the Falaco Solitons are related to strings connecting branes led to the thought that perhaps the modern advances in topology and string theory could yield a theoretical explanation of the formation and stability of Falaco Solitons. According, challenges and requests for help were sent out to many of the string theorists, asking for theoretical help to describe this "real life string connecting branes"; the lack of response indicates that none of the string gurus seemed to think the effort was worthwhile. However, the theoretical work of Dzhunushaliev [?] seems to have many correspondences with the experimental facts of the Falaco Solitons.

The Falaco Soliton endcap dimples (which are presumed to be surfaces of zero mean curvature and positive Gauss curvature) are related to Spinor eigen direction fields associated with antisymmetric matrices representing Symplectic spaces. If the Maximal surfaces appear as deformations in disconnected hypersurfaces of discontinuity, the topological structure has the appearance of "strings connecting branes", a concept touted by the string theorists .

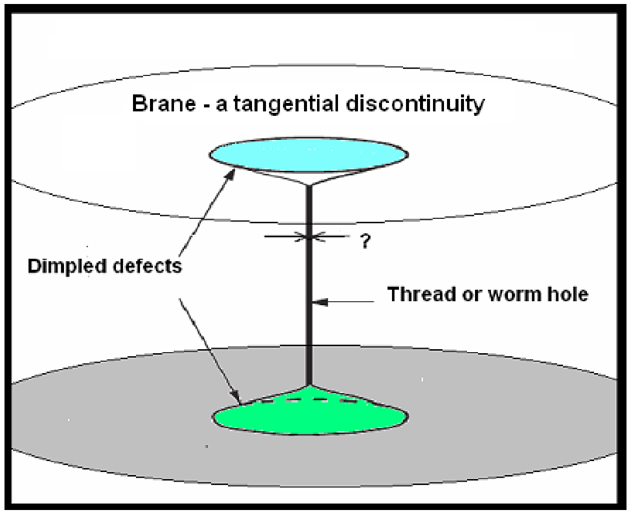


Figure 3.7 Falaco Solitons as connected dimples between Branes.

The new feature is that the "brane" surface of discontinuity is deformed by the Maximal surface dimple (which has been alluded to as a space-time foam [**?**]) This structure motivates the next section in which the idea is used to model the photon. The idea is also related to the rotational structures of rotating Bose-Einstein condensates [40].

3.4.1. Spinors and Zero Mean curvature surfaces.

The theory of minimal surfaces (of zero mean curvature) are intertwined with the concept of complex isotropic direction fields, defined as pure Spinors by E.Cartan. The Weirstrass formulas of minimal surface theory [24] consider a holomorphic complex velocity field in 3D, which upon integration leads to conjugate pairs of minimal surfaces defined by the real and imaginary components of the position vector formed by complex integration. The key feature of this holomorphic "velocity" field, so useful to minimal surface theory, is that it is a complex isotropic collection of components, whose Euclidean sums of squares is zero. Such isotropic complex direction fields of zero quadratic form (length) were defined as Spinors by

E. Cartan [11]. The theory of minimal surfaces draws attention to the relationship between Spinors and Harmonic vector fields, mentioned in the last subsection.

In addition, E. Cartan years ago demonstrated that infinitesimal rotations are generated by antisymmetric matrices. It is rather remarkable (and was only fully appreciated by the author in January, 2005) that there is a large class of direction fields S_4 representing thermodynamic processes that do not behave as diffeomorphic vectors . Such direction fields are Spinors and satisfy the equation,

$$\textbf{The Spinor Class:} \quad W = i(S_4)dA \neq 0, \tag{3.5}$$

Without going into mathematical detail, what this means, physically, is that there are thermodynamic processes that can be defined in terms of Spinors (not vectors) which produce thermodynamic work, but the value of the work done in the direction of the process is zero. In other words, the work done, W, is "transverse" to the to process generated by a Spinor. This concept distinguishes the concept of Heat, Q, from the concept of non-zero Work, W. Work is always transverse, Heat is not necessarily Transverse, but can have components associated with the process direction. Furthermore, non-zero Work is always created by Spinor direction fields, and therefor can have chiral effects.

Over the real domain, there are no "real vectors" that satisfy this quadratic form, but there are many complex vectors that satisfy the "isotropic" formula. In Euclidean 3 space, the complex integrals of the complex isotropic vectors, when separated into real and imaginary parts, lead to two conjugate 3D "position vectors" that describe immersions of minimal (zero mean curvature) surfaces in 3D.

Remark 5. *Falaco Solitons can represent non tensorial properties of Spinor analysis, and, lead to the possibility of surfaces of zero mean curvature, but with positive, not negative, Gauss curvature. Such results imply the existence of irreducible components of affine torsion (non-zero values of what is called topological torsion) and occur only in thermodynamic physical systems that are far from equilibrium.*

3.5. Falaco Solitons as Photons

The reader must remember that the Falaco Soliton is a topological object that can and will appear at all scales, from the microscopic, to the macroscopic, from the sub-submicroscopic world of strings connection branes, to the cosmological level of spiral arm galaxies connected by threads. At the microscopic level, the method offers a view of forming spin pairs that is different from Cooper pairs and could offer insight into Hi-TC Superconductivity. At the level of Cosmology, the concept of Falaco Solitons could lead to explanations of the formation of flat spiral arm galaxies. At the submicroscopic level, the Falaco Solitons mimic quark pairs confined by a string. At the microscopic level, the Falaco Solitons appear as the dimpled vortex structures in rotating Bose-Einstein condensates. They also model the concepts of a Photon as being the singular thread attached to dimples on two expanding light cone shells.

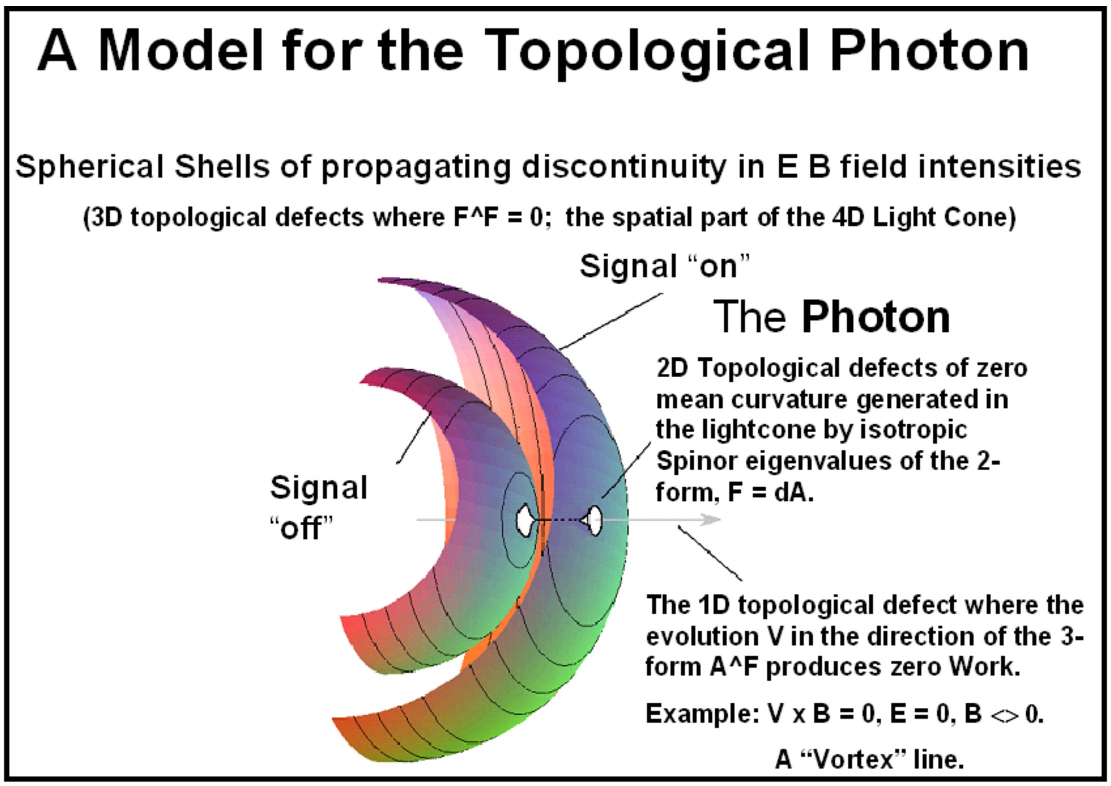


Figure 3.8

At the macroscopic level, similar topological features of the Falaco Solitons can be found in solutions to the Navier-Stokes equations in a rotating frame of reference. Under deformation of the discontinuity surface to a flattened ball, the visual correspondence to hurricane structures between the earth surface and the tropopause is remarkable. In short, as a topological defect, the concept of Falaco Solitons is a universal phenomenon valid at all scales.

3.6. Falaco Solitons as Landau Ginsburg structures in micro and mesoscopic systems

The Falaco experiments lead to the idea that such topological defects are available at all scales. In the microphysical world, the theory of phase transitions was developed on a more or less as hoc basis by Landau and Ginsburg. The theory has had application to problems in superconductivity, superfluids, and Bose Einstein condensates. The Falaco Solitons consist of spiral "vortex defect" structures (analogous to CGL theory) on a two dimensional minimal surface, one at each end of a 1-dimensional "vortex line" or thread (analogous to GPG theory). Remarkably the topological defect surface structure is locally unstable, as the surface is of negative Gauss curvature. Yet the pair of locally unstable 2-D surfaces is *globally* stabilized by the 1-D line defect attached to the "vertex" points of the minimal surfaces.

For some specific physical systems it can be demonstrated that period (circulation) integrals of the 1-form of Action potentials, A, lead to the concept of "vortex defect lines". The idea is extendable to "twisted vortex defect lines" in three dimensions. The "twisted vortex defects" become the spiral vortices of a Complex Ginsburg Landau (CGL) theory , while the "untwisted vortex lines" become the defects of Ginzburg-Pitaevskii-Gross (GPG) theory [39]. In the picture below, dimpled surfaces and the spiral arms are due to the work of Tornquist based on quantum mechanics and Landau - Ginsburg theory. The black spots and the connecting thread are my additions to the Torquist picture. It looks to me that Landau-Ginsburg theory has contact with Falaco Solitons.

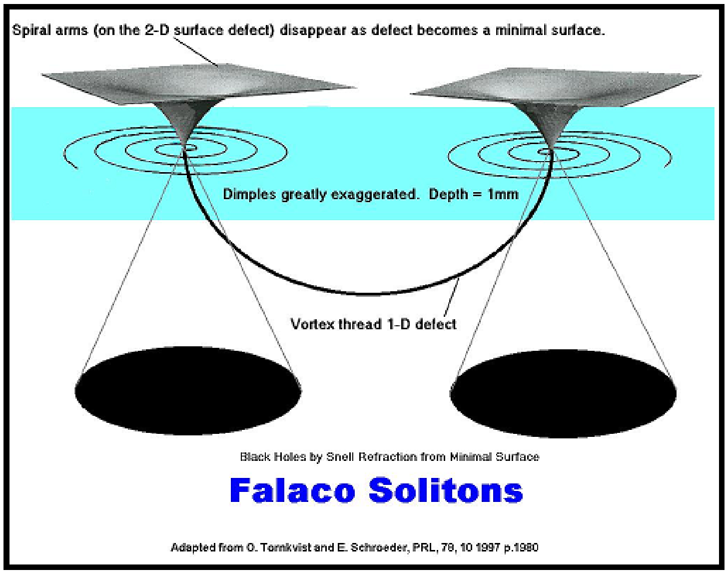


Figure 3.9

For super cold Bose Einstein condensates, the rotation defect structures have been described as U-shaped vortex singularities with dimples on the ends of the vortex "lines". The rotational minimal surfaces of Zero mean curvature, which form the two endcaps of the Falaco soliton, apparently are confined by the string, just like quarks. If the string (whose "tension" induces global stability of the unstable endcaps) is severed, the endcaps (like unconfined quarks in the elementary particle domain) disappear (in a non-diffusive manner). In the microscopic electromagnetic domain, the Falaco soliton structure offers an alternate, topological, pairing mechanism on a Fermi surface, that could serve as an alternate to the Cooper pairing in superconductors.

4. How to replicate the Experiment

The Falaco Soliton phenomenon is easily reproduced by placing a large circular disc, such as dinner plate, vertically into the swimming pool until the plate is half submerged and it oblate axis resides in the water-air free surface. Then move the plate slowly in the direction of its oblate axis. At the end of the stroke, smoothly extract the plate (with reasonable speed) from the water, imparting kinetic energy and distributed angular momentum to the fluid. The dynamical system undergoes a short period (a few seconds) of stabilization, followed by a longer period (many minutes) of a "topologically stationary" state. It is this topologically stationary state that is defined as the Falaco Soliton. Thermodynamically, the system starts in an initial state of Pfaff topological dimension 4 and decays by continuous topological evolution to a "stationary" state of Pfaff topological dimension 3. According to the theory of non equilibrium thermodynamics [41], the processes during the initial stabilization period are thermodynamically irreversible, but once the Pfaff dimension 3 configuration is reached, the evolutionary processes preserving topological features can be described in a Hamiltonian manner. Both the initial and the "stationary" soliton states are thermodynamic states far from equilibrium.

At first it was thought that the initial deformed surface state could be related to a Rankine vortex structure (which has regions of both positive and negative Gauss curvature). Recall that a Rankine vortex has a core that is equivalent to rigid body rotation. This description of the formative state of stabilization is too naive, for observations indicate that the sharp edge of the plate described above generates instability patterns [43] as it is stroked through the fluid. After the initial injection of energy and angular momentum, the fluid spends a few seconds during a process of stabilization, during which a surface of zero mean curvature is formed transiently, producing the easily visible large black spots formed by Snell refraction. Associated with the evolution to a "stationary" Soliton state, is a visible set of spiral arm caustics on the pool surface around each dimples rotation axis. As the stabilization proceeds, the spiral caustics appear to grow tighter around the black spot, and are almost gone when the Soliton becomes stable.

In a few tries you will become an expert experimentalist at stroking the plate and creating Falaco Solitons. The drifting black spots are easily created and, surprisingly, will persist for many minutes in a still pool. The dimpled depressions are typically of the order of a few millimeters in depth, but the zone of circulation around each rotation axis typically extends over a disc of some 10 to 30 centimeters radius, depending on the plate diameter. The "stationary" configuration, or coherent topological defect structure, has been defined as the Falaco Soliton. For purposes of illustration , the vertical depression has been greatly exaggerated in Figures 3 and 4.

If a thin broom handle or a rod is placed vertically in the pool, and the Falaco soliton pair is directed in its translation motion to intercept the rod symmetrically, as the soliton pair comes within range of the scattering center, or rod, (the range is approximately the separation distance of the two rotation centers) the large black spots at first shimmer and then disappear. Then a short time later, after the soliton has passed beyond the interaction range of the scattering center, the large black spots coherently reappear, mimicking the numerical simulations of soliton coherent scattering. For hydrodynamics, this observation firmly cements the idea that these objects are truly coherent "Soliton" structures. This experiment is the only (known to this author) macroscopic visual experiment that demonstrates these coherence features of soliton scattering.

If the string connecting the two endcaps is sharply "severed", the confined, two dimensional endcap singularities do not diffuse away, but instead disappear almost explosively. The process of "severing" can be accomplished by moving your hand (held under the water approximately above the circular arc or "string" connecting the two dimple vertices) in a karate chop motion. It is this observation that leads to the statement that the Falaco soliton is the macroscopic topological equivalent of the illusive hadron in elementary particle theory. The two 2-dimensional surface defects (the quarks) are bound together by a string of confinement, and cannot be isolated. The dynamics of such a coherent structure is extraordinary, for it is a system that is globally stabilized by the presence of the connecting 1-dimensional string.

For a movie of the process see [37].

5. Summary

As the Falaco phenomena appears to be the result of a topological defect, it follows that as a topological property of hydrodynamic evolution, it could appear in any density discontinuity, at any scale. This rotational pairing mechanism, as a topological phenomenon, is independent from size and shape, and could occur at both the microscopic and the cosmic scales. In fact, as mentioned above, during the formative stages of the Falaco Soliton pair, the decaying vortex structures exhibit spiral arms easily visible as caustics emanating from the boundary of each vortex core. The observation is so striking that it leads to the conjecture: Can the nucleus of M31 be connected to the nucleus of our Milky way galaxy by a tubular cosmic thread? Can material be ejected from one galaxy to another along this comic thread? Can barred spirals be Spiral Arm galaxies at an early stage of formation - the bar being and exhibition of material circulating about the stabilizing thread? At smaller scales, the concept also permits the development of another mechanism for producing spin-pairing of electrons in the discontinuity of the Fermi surface, or in two dimensional charge distributions. Could this spin pairing mechanism, depending on transverse wave, not longitudinal wave, coupling be another mechanism for explaining superconductivity? As the defect is inherently 3-dimensional, it must be associated with a 3-form of Topological Torsion, A^dA, introduced by the author in 1976 [?] [31] [33] [35], but now more commonly called a Chern Simons term, when applied to properties of a linear connection. These ideas were exploited in an attempt to explain high TC superconductivity [?]. To this author the importance of the Falaco Solitons is that they offer the first clean experimental evidence of topological defects taking place in a dynamical system. Moreover, the experiments are fascinating, easily replicated by anyone with access to a swimming pool, and stimulate thinking in almost everyone that observes them, no matter what his field of expertise. They certainly are among the most easily produced solitons.

6. Some Anecdotal History

Just at the end of WW II, one of my first contacts at MIT was a Brazilian young man named Jose Haraldo Hiberu FALCAO. He was in metallurgy and I was in physics. We became close friends and roommates during the period 1946-1950. He spent much of his time chasing the girls and playing soccer for MIT. Now MIT is not known for its athletic achievements, and when one weekend Haraldo scored two goals - giving MIT one of its few wins (ever) - the sports section of one of the Boston papers, misspelled his name with the headline ~

"FALACO SCORES TWO GOALS - MIT WINS"

Frankly I do not remember the exact headline from more than 55 years ago, but one thing is sure: Haraldo FALCAO was known as FALACO ever since.

Haraldo moved back to Brazil and our ways parted. I became interested in many things, the most pertinent to this story included topological defects and topological evolution in physical systems. In 1986 I thought it would be great fun to go to Rio to see my old college friend, and then go to Machu Pichu to watch Haley's comet go by. My son was an AA pilot, so as parents we got a free Airline Ticket ticket to Brazil. Haraldo had married into a very wealthy family and had constructed a superb house that his wife had designed, hanging onto a cliff-side above Sao Coronado beach south of Rio. Haraldo had a white marble swimming pool next to the house fed by a pristine stream of clear water.

The morning after my wife and I arrived in Rio (Haraldo's chauffeur met us at the airport in a big limo) I got up, after sleeping a bit late, and went to the pool for a morning dip. Haraldo and his family had gone to work, and no one was about. I sat in the pool, wondering about the fortunes of life, and how Haraldo - who I had help tutor to get through MIT - was now so very wealthy, and here I was - just a *pauvre* university professor. I climbed out of the pool, and was met by two servants who had been waiting in the wings. One handed me a towel and a terry cloth robe, and the other poured coffee and set out some breakfast fruit, croissants, etc.

I put a lot of sugar into the strong Brazilian coffee, as Haraldo had taught me to do long ago, and was stirring the coffee when I turned toward the pool (about 5-10 minutes after climbing out of the pool). In the otherwise brilliant sunshine, two black disks (about 15 cm in diameter) with bright halo rings were slowing translating along the pool floor. The optics caught my attention. Is there something about the southern hemisphere that is different? Does the water go down the drain with a different rotation? What was the cause of these Black Discs?

I went over to the pool, jumped in to investigate what was going on, and Voila!!!, the black discs disappeared. I thought: Here was my first encounter of the third kind and I blew it.

I climbed out of the pool, again, and then noticed that a pair of what I initially thought to be Rankine vortices was formed as my hips left the water, and that these rotational surfaces (which would be surface depressions of positive Gauss curvature if they were Rankine vortex structures) decayed within a few seconds into a pair of rotational surfaces of negative Gauss curvature. Each of the ultimate rotational surfaces were as if someone had depressed slightly a rubber sheet with a pencil point, forming a dimple. As the negative Gauss curvature surfaces stabilized, the optical black disks were formed on the bottom of the pool. The extraordinary thing was that the surface deformations, and the black spots, lasted for some 15 minutes !!!. They were obviously rotational solitons.

The rest is history, and is described on my website and in several published articles in some detail. The first formal presentation was at the 1987 Austin Dynamic Days get together, where my presentation and photos cause quite a stir. The Black Discs were quickly determined to be just an artifact of Snell's law of refraction of the solar rays interacting with the dimpled surfaces of negative Gauss curvature. I conjectured that this Soliton was a topological defect, which caused the mathematicians to take note. It was then that I met Dennis Sullivan, who many years later, along with Bobenko, would influence me again with the concept that minimal surfaces and spinors were related ideas.

What was not at first apparent in the swimming pool experiment was that there is a circular "string" – a 1D topological defect – that connects the two 2D topological defects of negative Gauss curvature. The string extends from one dimple to the other. The string becomes evident if you add a few drops of dye to the water near the rotation axis of one of the "dimples". Moreover, experimentation indicated that the long term soliton stability was due to the global effect of the "string" connecting the two dimpled rotational surfaces. If the arc is sharply severed, the dimples do not "ooze" away, as you would expect from a diffusive process; instead they disappear quite abruptly. It startled me to realize that the Falaco Solitons have the confinement properties (and problems) of two quarks on the end of a string.

I called the objects FALACO SOLITONS, for they came to my attention in Haraldo's pool in Rio. Haraldo will get his place in history. I knew that finally I had found a visual, easily reproduced, experiment that could be used to show people the importance and utility of Topological Defects in the physical sciences, and could be used to promote my ideas of Continuous Topological Evolution.

The observations were highly motivating. The experimental observation of the Falaco Solitons greatly stimulated me to continue research in applied topology, involving topological defects, and the topological evolution of such defects which can be associated with phase changes and thermodynamically irreversible and turbulent phenomena. When colleagues in the physical and engineering sciences would ask "What is a topological defect?" it was possible for me to point to something that they could replicate and understand visually at a macroscopic level.

The topological ideas have led ultimately to

1. A non-statistical method of describing processes that are thermodynamically irreversible.

2. Applications of Topological Spin and Topological Torsion in classical and quantum field theories.

3. Another way of forming Fermion pairs

4. A suggestion that spiral galaxies may be stabilized by a connecting "thread", and an explanation of the fact that stars in the far reaches of galactic spiral arms do not obey the Kepler formula.

5. A number of patentable ideas in fluids, electromagnetism, and chemistry.

The original observation was first described at a Dynamics Days conference (1987) in Austin, TX, [27] and has been reported, as parts of other research, in various hydrodynamic publications, but it is apparent that these concepts have not penetrated into other areas of research. As the phenomena is a topological defect, and can happen at all scales, the Falaco Soliton should be a natural artifact of both the sub-atomic and the cosmological worlds. The reason d'etre for this chapter is to bring the idea to the attention of other researchers who might find the concept of Falaco Solitons interesting and stimulating to their own research.

7. References

References

[1] Brand, L. (1962) "Vector and Tensor Analysis", Wiley, N.Y. p. 172-173

[2] Brillouin, L. (1964) "Tensors in Mechanics and Elasticity", Academic Press, NY p 93. In 1938, Brillouin wrote: "If one does not admit the symmetry of the (connection) coefficients, Γ_{ijk}, one obtains the twisted spaces of Cartan, spaces which scarcely have been used in physics to the present, but which seem to be called to an important role."

[3] This author was alerted to the Brouwer theorem by Flanders, H. (1963) "Differential Forms" Academic, New York,, p. 72; the theorem is given in many texts.

[4] Bryant, R.L.,Chern, S.S., Gardner, R.B.,Goldschmidt, H.L., and Griffiths, P. A. (1991) "Exterior Differential Systems", Springer, Verlag.

[5] Caratheodory, C. (1909) Math. Ann. 67, 355

[6] Carnevale, G. F. and Vallis, G. K. (1990) Isovortical Relaxation to Stable Two-Dimensional Flows in: "Topological Fluid Mechanics", H. K. Moffatt and T. S. Tsinober eds, Cambridge University Press, Cambridgep 449-458. Also see Carnevale, G. F. and Vallis, G. K. (1989) J. Fluid Mech., 207 p.133-152

[7] Cartan, E. (1899) "Sur certaines expressions differentielles et le systeme de Pfaff" Ann Ec. Norm. 16 329

[8] Cartan, E. (1922) "Systems Differentials Exterieurs et leurs Applications Geometriques" Hermann, Paris.

[9] Cartan, E. (1937) "La Theorie des Spaces a Connexion Projective", Hermann, Paris

[10] Cartan, E., (1958) "Lecons sur les invariants integrauxs", Hermann, Paris.

[11] Cartan, E. (1966) "The Theory of Spinors", Dover, New York, p. 39

[12] Eckart, C., (1940), "The thermodynamics of irreversible processes I: The simple fluid, II: Fluid mixtures, III: Relativistic theory of the simple fluid", Phys. Rev., 58,

[13] Eddington, A. S. (1963) "The Mathematical Theory of Relativity", Cambridge University Press, p. 194.

[14] Eisenhart, L. P. (1963) "Continuous Groups", Dover, N. Y.A route to chaos and turbulence" in Advances in Fluid Turbulence, edited by G. Doolen, R, Ecke, D. Holm and V. Steinberg, (North Holland, Amsterdam) p. 116.an Nostrand, London.

[15] Fernandez, I., Lopez, F. J., Souam, R. "The space of complete embedded maximal surfaces with isolated singularities in the 3-dimensional Lorentz-Minkowski space L^3", arXiv:math.DG/0311330 Rev v2 27 JAN 2005

[16] Landau, L. D. and Liftshitz, E. M. (1959) "Fluid Mechanics", Pergamon p. 94.

[17] Landau, L. D. and Lifshitz, E. M., (1958), "Statistical Physics" Pergamon Press, London p. 369.

[18] Landau, L. D. and Lifshitz, E. M. (1960), "Electrodynamics of Continuous Media", Pergamon Press, London, p. 97.

[19] Landsberg, P. T. (1961), "Thermodynamics" Interscience NY

[20] Lang, S. (1962), "Introduction to Differentiable Manifolds" Interscience, New York.

[21] Langford, W. F. (1983) in " non linear Dynamics and Turbulence" Edited by G.I. Barrenblatt, et. al. Pitman, London

[22] Marsden, J.E. and Riatu, T. S. (1994) "Introduction to Mechanics and Symmetry", Springer-Verlag, p.122

[23] http://www22.pair.com/csdc/pdf/maxlor.pdf

[24] Osserman, R. (1986) "A Survey of Minimal Surfaces", Dover, N.Y. 1986)

[25] deRham, G. (1960) "Varietes Differentiables", Hermann, Paris.

[26] The importance of the N-1 form A^H (now written as A^G) was first anticipated in: R. M. Kiehn and J. F. Pierce, *An Intrinsic Transport Theorem* Phys. Fluids 12, 1971. The concept was further employed in R. M. Kiehn, *Periods on manifolds, quantization and gauge*, J. of Math Phys 18 no. 4 p. 614 (1977), and with applications described in R. M. Kiehn, *Are there three kinds of superconductivity,* Int. J. Mod. Phys B 5 1779. (1991)

[27] Kiehn, R. M., (1987) The Falaco Effect as a topological defect was first noticed by the present author in the swimming pool of an old MIT friend, during a visit in Rio de Janeiro, at the time of Halley's comet, March 1986. The concept was presented at the Austin Meeting of Dynamic Days in Austin, January 1987, and caused some interest among the resident topologists. The easily reproduced experiment added to the credence of topological defects in fluids. It is now perceived that this topological phenomena is universal, and will appear at all levels from the microscopic to the galactic. LANL arXiv/gr-qc/0101098 (../falaco97.pdf and../topturb.pdf)

[28] Sterling. et.al. (1987), Why are these disks dark? Phys Fluids 30 p.11

[29] Kiehn, R. M., (1989), Irreversible Topological Evolution in Fluid Mechanics in "Some Unanswered Questions in Fluid Mechanics" ASME- Vol. 89-WA/FE-5, Trefethen, L. M. and Panton, R. L. Eds.

[30] Kiehn, R.M, (1987) "The Falaco Effect, A Topological Soliton" Talk presented at the Dynamics Days conference, Austin, TX Jan. 1987. The Falaco Effect as a topological defect was first noticed by the present author in the swimming pool of an old MIT friend, during a visit in Rio de Janeiro, at the time of Halley's comet, March 1986. The concept was presented at the Austin Meeting of Dynamic Days in Austin, January 1987, and caused some interest among the resident topologists. The easily reproduced experiment added to the credence of topological defects in fluids. It is now perceived that

this topological phenomena is universal, and will appear at all levels from the microscopic to the galactic.

[31] Kiehn, R.M. (1990), Topological Torsion, Pfaff Dimension and Coherent Structures, in: "Topological Fluid

[32] Kiehn, R. M., (1999), Coherent Structures in Fluids are Topological Torsion Defects, in J, "IUTAM Symposium on Simulation and Identification of Organized Structures in Flows",.N. Sørensen, et al., eds., Kluwer Academic Publishers, Dordrecht,. See (../copen5.pdf). Presented at the IUTAM-SIMFLO Conference at DTU, Denmark, May 25-29, (1997).

[33] Kiehn, R. M. (1991), Compact Dissipative Flow Structures with Topological Coherence Embedded in Eulerian Environments, in: " non linear Dynamics of Structures", edited by R.Z. Sagdeev, U. Frisch, F. Hussain, S. S. Moiseev and N. S. Erokhin, p.139-164, World Scientific Press, Singapore

[34] Kiehn, R.M. (1991) "Continuous Topological Evolution", LANL arXiv/math-ph/0101032 (../contevol3.pdf)

[35] Kiehn, R. M. (1992), Topological Defects, Coherent Structures and Turbulence in Terms of Cartan's Theory of Differential Topology, in "Developments in Theoretical and Applied Mathematics, Proceedings of the SECTAM XVI conference", B. N. Antar, R. Engels, A.A. Prinaris and T. H. Moulden, Editors, The University of Tennessee Space Institute, Tullahoma, TN 37388 USA.

[36] Rodriguez, C. C. (1991) From Euclid to Entropy, In *Maximum Entropy and Bayesian Methods*, W. T. Grandy, Jr. Ed., Kluwer Academic Publishers, pp 1-6

[37] http://www22.pair.com/csdc/download/spotsmovie.avi

[38] Struik, D. (1961) "Differential Geometry", Addison Wesley, Reading, Mass

[39] Tornkvist, O. and Schroeder, E., (1997) Vortex dynamics in Dissipative Systems, Phys Rev. Lett. 78, 10, p. 1908-1911.

[40] Volovik, G. E., (2003) "The Universe in a Helium droplet", Claredon Press, Oxford, 2003

[41] Kiehn, R. M. (2004) Non Equilibrium Thermodynamics, "Non Equilibrium Systems and Irreversible Processes Vol 1" see http://www.cartan.pair.com

[42] Kiehn, R. M. (2004) Cosmology, Falaco Solitons and the Arrow of Time, "Non Equilibrium Systems and Irreversible Processes Vol 2" see http://www.cartan.pair.com

[43] Kiehn, R. M. (2004) Wakes, Coherent Structures and Turbulence, "Non Equilibrium Systems and Irreversible Processes Vol 3" see http://www.cartan.pair.com

[44] Kiehn, R. M. (2004) Plasmas and Non equilibrium Electrodynamics, "Non Equilibrium Systems and Irreversible Processes Vol 4" see http://www.cartan.pair.com

[45] Kiehn, R. M. (2004) Exterior Differential Forms and Differential Topology, "Non Equilibrium Systems and Irreversible Processes Vol 5" see http://www.cartan.pair.com

[46] Kiehn, R. M. (2004) Maple Magic and Differential Topology "Non Equilibrium Systems and Irreversible Processes Vol 6" see http://www.cartan.pair.com

[47] Zeldovish, Ya B. and Novikov, I. D. (1996) " Stars and Relativity", Dover, New York) p.273-288.